

# BRST, anti-BRST and gerbes

L. Bonora, R. P. Malik<sup>1</sup>

*International School for Advanced Studies (SISSA),  
Strada Costiera, Via Beirut n.2-4, 34013 Trieste, Italy*  
and

*INFN, Sezione di Trieste, Trieste, Italy*

**E-mails:** bonora@he.sissa.it; malik@bhu.ac.in

**Abstract:** We discuss BRST and anti-BRST transformations for an Abelian antisymmetric gauge field in 4D and find that, in order for them to anticommute, we have to impose a condition on the auxiliary fields. This condition is similar to the Curci-Ferrari condition for the 4D non-Abelian 1-form gauge theories and represents a consistency requirement. We interpret it as a signal that our Abelian 2-form gauge field theory is based on gerbes. To support this interpretation we discuss, in particular, the case of the 1-gerbe for our present field theory and write the relevant equations and symmetry transformations for 2-gerbes.

PACS numbers: 11.15.-q, 12.20.-m, 03.70.+k

**Keywords:** Free Abelian 2-form gauge theory, anticommuting (anti-)BRST symmetries, analogue of the Curci-Ferrari condition, gerbes

---

<sup>1</sup>On leave of absence from Centre of Advanced Studies, Physics Department, Faculty of Science, Banaras Hindu University, Varanasi-221 005 (U.P.), India.

# 1 Introduction

While the BRST symmetry has been a fundamental tool for the study of quantum field theories in the last three decades, the anti-BRST symmetry does not seem to have played more than a decorative role. In this paper, we would like to suggest that, perhaps, the system of BRST and anti-BRST symmetry contains more information than it is generally attributed to it. To start with, we discuss some features of BRST and anti-BRST transformations in theories of free Abelian two-form  $B_{\mu\nu}$  fields and show that the requirement that the BRST and anti-BRST transformations must anti-commute, imposes a constraint on the fields of the theory, very similar to the one that must be imposed for the 4D 1-form non-Abelian gauge theories (i.e. the Curci-Ferrari condition). The idea we want to develop in this paper is that this type of constraints is characteristic, not only of the non-Abelian gauge theories, but also of higher form Abelian gauge theories whose field content is based on gerbes. To clarify this point, we first give a brief introduction to the subject of the Abelian gerbes. Then, we find the BRST and anti-BRST transformations for the 1-gerbe and show that their anti-commutativity requires precisely constraints of the above type. We show also that the 2-gerbes require two such constraints. We interpret all this as evidence that, indeed, such type of Curci-Ferrari constraints are characteristic of the gerbe-based field theory.

The contents of the paper are organized as follows. In section 2, we discuss the bare essentials of the nilpotent but non-anti-commuting symmetry transformations for the free Abelian 2-form gauge theory in 4D. This is followed, in section 3, by the discussion and derivation of the nilpotent and anti-commuting (anti-)BRST symmetry transformations for the above theory. Section 4 deals with the brief synopsis of the Abelian gerbes. The material of section 5 concerns the anti-commutativity property of the (anti-)BRST symmetry transformations and its connection with the gerbes. Finally, we make some concluding remarks in section 6.

## 2 Nilpotent and non-anticommuting symmetry transformations: a brief synopsis

We begin with the following nilpotent symmetry invariant Lagrangian density for the 4D free Abelian 2-form gauge theory [1,2,3]

$$\begin{aligned} \mathcal{L}_b = & \frac{1}{12} H^{\mu\nu\kappa} H_{\mu\nu\kappa} + B^\mu (\partial^\nu B_{\nu\mu} - \partial_\mu \phi) - \frac{1}{2} B^\mu B_\mu - \partial_\mu \bar{\beta} \partial^\mu \beta \\ & + (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu) (\partial^\mu C^\nu) + \rho (\partial \cdot C + \lambda) + (\partial \cdot \bar{C} + \rho) \lambda. \end{aligned} \quad (1)$$

In the above, the kinetic energy term is constructed with the curvature tensor  $H_{\mu\nu\kappa}$  which is an intrinsic component of the three-form  $H^{(3)} = dB^{(2)} = (1/3!)(dx^\mu \wedge dx^\nu \wedge dx^\kappa)H_{\mu\nu\kappa}$  where 2-form  $B^{(2)} = (1/2!)(dx^\mu \wedge dx^\nu)B_{\mu\nu}$  defines the gauge potential  $B_{\mu\nu}$  of the theory<sup>2</sup>. The Nakanishi-Lautrup auxiliary vector field  $B_\mu$  is invoked to linearize the gauge-fixing term  $[(1/2)(\partial^\nu B_{\nu\mu} - \partial_\mu \phi)^2]$ . The latter requires, for the nilpotent symmetry invariance in the theory, the fermionic vector (anti-)ghost fields  $(\bar{C}_\mu)C_\mu$  as well as the bosonic (ghost-for-ghost) fields  $(\bar{\beta})\beta$ . The above symmetry invariant Lagrangian density also requires fermionic auxiliary ghost fields  $\rho$  and  $\lambda$  (for the gauge-fixing of the vector (anti-)ghost fields) and a massless (i.e.  $\square\phi = 0$ ) scalar field  $\phi(x)$  for the stage-one reducibility that is present in the Abelian 2-form gauge theory.

The following off-shell nilpotent, local, covariant, continuous and infinitesimal transformations<sup>3</sup>

$$\begin{aligned}\tilde{s}_b B_{\mu\nu} &= -(\partial_\mu C_\nu - \partial_\nu C_\mu), & \tilde{s}_b C_\mu &= -\partial_\mu \beta, & \tilde{s}_b \bar{C}_\mu &= -B_\mu, \\ \tilde{s}_b \phi &= \lambda, & \tilde{s}_b \bar{\beta} &= -\rho, & \tilde{s}_b[\rho, \lambda, \beta, B_\mu, H_{\mu\nu\kappa}] &= 0,\end{aligned}\quad (2)$$

$$\begin{aligned}\tilde{s}_{ab} B_{\mu\nu} &= -(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu), & \tilde{s}_{ab} \bar{C}_\mu &= +\partial_\mu \bar{\beta}, & \tilde{s}_{ab} C_\mu &= +B_\mu, \\ \tilde{s}_{ab} \phi &= \rho, & \tilde{s}_{ab} \beta &= -\lambda, & \tilde{s}_{ab}[\rho, \lambda, \bar{\beta}, B_\mu, H_{\mu\nu\kappa}] &= 0,\end{aligned}\quad (3)$$

leave the above Lagrangian density (1) quasi-invariant because it transforms as:  $\tilde{s}_b \mathcal{L}_b = -\partial_\mu [B^\mu \lambda + (\partial^\mu C^\nu - \partial^\nu C^\mu) B_\nu - \rho \partial^\mu \beta]$  and  $\tilde{s}_{ab} \mathcal{L}_b = -\partial_\mu [B^\mu \rho + (\partial^\mu \bar{C}^\nu - \partial^\nu \bar{C}^\mu) B_\nu - \lambda \partial^\mu \bar{\beta}]$ . These transformations have been christened as the (anti-)BRST symmetry transformations  $\tilde{s}_{(a)b}$  for the 4D free Abelian 2-form gauge theory [2,3]. However, there is one key property that is *not* satisfied by the above nilpotent symmetry transformations: the BRST ( $\tilde{s}_b$ ) and anti-BRST ( $\tilde{s}_{ab}$ ) transformations do not anti-commute (as is the case, for instance, with the true (anti-)BRST symmetry transformations found in the case of any arbitrary gauge (or reparametrization) invariant theories that are endowed with the first-class constraints in the language of the Dirac's prescription for the classification scheme [5,6]).

In fact, while this is true for most of the local fields, present in the Lagrangian density (1), namely;

$$\{\tilde{s}_b, \tilde{s}_{ab}\} \Phi(x) = 0, \quad \Phi(x) = B_{\mu\nu}, B_\mu, \beta, \bar{\beta}, \lambda, \rho, \quad (4)$$

it can be readily checked, from (2) and (3), that  $\tilde{s}_{ab} \tilde{s}_b \bar{C}_\mu = 0$ ,  $\tilde{s}_b \tilde{s}_{ab} C_\mu = 0$  but  $\tilde{s}_b \tilde{s}_{ab} \bar{C}_\mu = -\partial_\mu \rho \neq 0$ ,  $\tilde{s}_{ab} \tilde{s}_b C_\mu = +\partial_\mu \lambda \neq 0$ . As a consequence, we find

---

<sup>2</sup>We follow here the convention and notations for the 4D Minkowski spacetime manifold with the metric  $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$  where the Greek indices  $\mu, \nu, \dots = 0, 1, 2, 3$ .

<sup>3</sup>It will be noted that these nilpotent transformations are same as the ones given in [4]. These differ from our earlier choice of the same [2,3] by a sign factor.

that  $\{\tilde{s}_b, \tilde{s}_{ab}\}C_\mu \neq 0$  and  $\{\tilde{s}_b, \tilde{s}_{ab}\}\bar{C}_\mu \neq 0$ . In the literature, it has been mentioned that the above transformations are anticommuting modulo the gauge transformations (i.e.  $\{\tilde{s}_b, \tilde{s}_{ab}\}C_\mu = -\partial_\mu\rho$  and  $\{\tilde{s}_b, \tilde{s}_{ab}\}\bar{C}_\mu = +\partial_\mu\lambda$ ) (see, e.g. [2]). However, this unpleasant aspect can be avoided, as we shall see in the next section.

The gauge-fixing and Faddeev-Popov ghost terms of the Lagrangian density (1) can be separately written as

$$\tilde{s}_b \left[ -\bar{C}^\mu \{(\partial^\nu B_{\nu\mu} - \partial_\mu\phi) - \frac{1}{2}B_\mu\} - \bar{\beta}(\partial \cdot C + 2\lambda) \right], \quad (5)$$

and

$$\tilde{s}_{ab} \left[ +C^\mu \{(\partial^\nu B_{\nu\mu} - \partial_\mu\phi) - \frac{1}{2}B_\mu\} + \beta(\partial \cdot \bar{C} + 2\rho) \right]. \quad (6)$$

The above expressions provide a simple and straightforward proof for the nilpotent symmetry invariance of the Lagrangian density (1) because of (i) the nilpotency of the transformations  $\tilde{s}_{(a)b}$ , and (ii) the invariance of the curvature term (i.e.  $\tilde{s}_{(a)b}H_{\mu\nu\kappa} = 0$ ) under  $\tilde{s}_{(a)b}$ . However the above gauge-fixing and Faddeev-Popov ghost terms can *never* be expressed as the BRST ( $\tilde{s}_b$ ) and anti-BRST ( $\tilde{s}_{ab}$ ) exact form *together* because of the non-anticommutativity property of the above nilpotent transformations.

### 3 Nilpotent and anticommuting (anti-)BRST symmetry transformations

Here we will show that the previous drawback can be fixed by introducing a constrained auxiliary field. It can be seen that the following off-shell nilpotent (i.e.  $s_{(a)b}^2 = 0$ ) (anti-)BRST symmetry transformations  $s_{(a)b}$

$$\begin{aligned} s_b B_{\mu\nu} &= -(\partial_\mu C_\nu - \partial_\nu C_\mu), & s_b C_\mu &= -\partial_\mu \beta, & s_b \bar{C}_\mu &= -B_\mu, \\ s_b \phi &= \lambda, & s_b \bar{\beta} &= -\rho, & s_b \bar{B}_\mu &= -\partial_\mu \lambda, & s_b [\rho, \lambda, \beta, B_\mu, H_{\mu\nu\kappa}] &= 0, \end{aligned} \quad (7)$$

$$\begin{aligned} s_{ab} B_{\mu\nu} &= -(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu), & s_{ab} \bar{C}_\mu &= -\partial_\mu \bar{\beta}, & s_{ab} C_\mu &= +\bar{B}_\mu, \\ s_{ab} \phi &= \rho, & s_{ab} \beta &= -\lambda, & s_{ab} B_\mu &= +\partial_\mu \rho, & s_{ab} [\rho, \lambda, \bar{\beta}, \bar{B}_\mu, H_{\mu\nu\kappa}] &= 0, \end{aligned} \quad (8)$$

are *anticommuting* (i.e.  $(s_b + s_{ab})^2 \equiv \{s_b, s_{ab}\} = 0$ ) in nature if the whole 4D free Abelian 2-form gauge theory is defined on a constrained surface parametrized by the following field equation

$$B_\mu - \bar{B}_\mu - \partial_\mu \phi = 0. \quad (9)$$

In fact, the anti-commutator  $\{s_b, s_{ab}\}B_{\mu\nu} = 0$  is valid only if the above equation (9) is satisfied. Moreover, it is straightforward to check that  $\{s_b, s_{ab}\}C_\mu = 0$  and  $\{s_b, s_{ab}\}\bar{C}_\mu = 0$  which were not true for the nilpotent symmetry transformations (2) and (3) discussed in the previous section.

The above equation (9) is the analogue of the Curci-Ferrari restriction [7] which is defined in the context of proving the anti-commutativity property (i.e.  $\{s_b, s_{ab}\} = 0$ ) of the nilpotent (anti-)BRST symmetry transformations  $s_{(a)b}$  for the 4D non-Abelian 1-form gauge theory. Furthermore, it can be checked, by exploiting the equations (7) and (8), that the condition (9) remains invariant under the anti-commuting (anti-)BRST symmetry transformations (i.e.  $s_{(a)b}[B_\mu - \bar{B}_\mu - \partial_\mu\phi] = 0$ ). The reason behind the existence of the constrained field equation (9), for the 4D Abelian 2-form gauge theory, comes from the superfield approach to BRST formalism [4].

We can express (9) as an equation of motion from a single Lagrangian density. This can be done if we introduce a Lagrange multiplier field  $L_\mu$  in an appropriate BRST invariant Lagrangian density in the following manner

$$\begin{aligned}\mathcal{L}^{(b)} &= \frac{1}{12}H^{\mu\nu\kappa}H_{\mu\nu\kappa} + B^\mu(\partial^\nu B_{\nu\mu}) + \frac{1}{2}(B \cdot B + \bar{B} \cdot \bar{B}) \\ &+ \partial^\mu \bar{\beta} \partial_\mu \beta - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu)(\partial^\mu C^\nu) \\ &+ (\partial \cdot C - \lambda)\rho + (\partial \cdot \bar{C} + \rho)\lambda + L^\mu(B_\mu - \bar{B}_\mu - \partial_\mu \phi),\end{aligned}\quad (10)$$

where the multiplier field  $L_\mu$  transforms under BRST transformation as:  $s_b L_\mu = -\partial_\mu \lambda$ . This is consistent with the equations of motion w.r.t.  $B_\mu, L_\mu, \bar{B}_\mu, \phi$ , derived from the above Lagrangian density, as given below

$$\begin{aligned}\partial^\nu B_{\nu\mu} + B_\mu + L_\mu &= 0, & \bar{B}_\mu - L_\mu &= 0, \\ B_\mu - \bar{B}_\mu - \partial_\mu \phi &= 0, & \square \phi + \partial_\mu L^\mu &= 0.\end{aligned}\quad (11)$$

The above equations, ultimately, imply  $\partial \cdot B = 0, \partial \cdot \bar{B} = 0, \square \phi = 0, L_\mu = \bar{B}_\mu$ . The transformation  $s_b L_\mu = -\partial_\mu \lambda$  is consistent with  $L_\mu = \bar{B}_\mu$  if we compare it with the BRST transformations (7) under which the Lagrangian density (10) transforms as:  $s_b \mathcal{L}^{(b)} = -\partial_\mu [(\partial^\mu C^\nu - \partial^\nu C^\mu)B_\nu + \lambda B^\mu + \rho \partial^\mu \beta]$ .

It is worthwhile to point out that the gauge-fixing and Faddeev-Popov ghost terms of the Lagrangian density (10) have been obtained by exploiting the anti-commuting (anti-)BRST symmetry transformations (7) and (8) as

$$\begin{aligned}s_b s_{ab} \left[ 2\beta \bar{\beta} + \bar{C}_\mu C^\mu - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \right] &= B^\mu(\partial^\nu B_{\nu\mu}) + B \cdot \bar{B} + \partial_\mu \bar{\beta} \partial^\mu \beta \\ &+ (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu)(\partial^\mu C^\nu) + (\partial \cdot C - \lambda)\rho + (\partial \cdot \bar{C} + \rho)\lambda.\end{aligned}\quad (12)$$

We have used the constraint field equation (9) to express

$$B \cdot \bar{B} = \frac{1}{2}(B \cdot B + \bar{B} \cdot \bar{B}) - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi. \quad (13)$$

It should be noted that (12) cannot be obtained in terms of the nilpotent transformations (2) and (3) which are non-anticommuting in nature.

Similarly, we can write the anti-BRST invariant Lagrangian density as:

$$\begin{aligned} \mathcal{L}^{(ab)} &= \frac{1}{12}H^{\mu\nu\kappa}H_{\mu\nu\kappa} + \bar{B}^\mu(\partial^\nu B_{\nu\mu}) + \frac{1}{2}(B \cdot B + \bar{B} \cdot \bar{B}) \\ &+ \partial^\mu \bar{\beta} \partial_\mu \beta - \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu)(\partial^\mu C^\nu) \\ &+ (\partial \cdot C - \lambda)\rho + (\partial \cdot \bar{C} + \rho)\lambda + L^\mu(B_\mu - \bar{B}_\mu - \partial_\mu \phi). \end{aligned} \quad (14)$$

Note that, only in the second term of the BRST invariant Lagrangian density (10), we have changed  $B_\mu \rightarrow \bar{B}_\mu$  which is consistent with (9). The equations of motion, derived from the  $\mathcal{L}^{(ab)}$ , are

$$\begin{aligned} \partial^\nu B_{\nu\mu} + \bar{B}_\mu - L_\mu &= 0, & B_\mu + L_\mu &= 0, \\ B_\mu - \bar{B}_\mu - \partial_\mu \phi &= 0, & \square \phi + \partial_\mu L^\mu &= 0. \end{aligned} \quad (15)$$

We derive, from the above, the equations  $\partial \cdot B = 0, \partial \cdot \bar{B} = 0, \square \phi = 0, L_\mu = -B_\mu$ . The anti-BRST symmetry transformation  $s_{ab}L_\mu = -\partial_\mu \rho$  for the Lagrange multiplier field is consistent with  $B_\mu + L_\mu = 0$  and the transformations (8). Under the latter nilpotent transformations, the Lagrangian density (14) transforms as:  $s_{ab}\mathcal{L}^{(ab)} = -\partial_\mu[(\partial^\mu \bar{C}^\nu - \partial^\nu \bar{C}^\mu)\bar{B}_\nu - \rho \bar{B}^\mu + \lambda \partial^\mu \bar{\beta}]$ . The constraint equation (9) emerges, as an equation of motion, from both the Lagrangian densities (10) as well as (14) which are equivalent and (anti-)BRST invariant on the constrained surface defined by the field equation (9).

## 4 Gerbes

The constraint (9) is intriguing. A similar type of constraint appears in the non-Abelian 1-form gauge theories when we implement the requirement of the anticommutativity of the BRST and anti-BRST transformations. The latter was introduced first by Curci and Ferrari [7] and was definitely related to the non-Abelian structure of the theory (see [11] where the Curci-Ferrari condition was embedded in the appropriate geometrical context). In the present case, the structure of the gauge transformations are definitely Abelian. Therefore the presence of this constraint calls for a definite novel motivation. We would like to suggest, in this context, that the rationale

behind (9) is not to be traced back to the non-Abelianity of the theory but, rather, to an underlying gerbe structure in the theory represented by the Lagrangian densities (10) or (14).

Gerbes form a hierarchy of geometrical structures (over space-time  $M$ ) whose simplest instance is a line bundle, or 0-gerbe (for a mathematical introduction see [12, 13, 14], for physical applications see [15] and references therein). The next more complicated case, in the above hierarchy, is a 1-gerbe. This is roughly speaking a ‘local’ line bundle. The latter is the assignment of a line bundle for each patch of a covering of  $M$ , for which a cocycle condition is required for the quadruple intersections (rather than for triple ones, which characterizes line bundles).

A 1-gerbe may be characterized by a triple  $(B, A, f)$ , formed by the 2-forms  $B$ , 1-forms  $A$  and 0-forms  $f$ , respectively <sup>4</sup>. These are related in the following way. Given a covering  $\{U_i\}$  of  $M$ , we associate to each  $U_i$  a two-form  $B_i$ . On a double intersection  $U_i \cap U_j$ , we have  $B_i - B_j = dA_{ij}$ . On the triple intersections  $U_i \cap U_j \cap U_k$ , we must have  $A_{ij} + A_{jk} + A_{ki} = df_{ijk}$ . Finally, on the quadruple intersections  $U_i \cap U_j \cap U_k \cap U_l$ , the following integral cocycle condition must be satisfied:

$$f_{ijl} - f_{ijk} + f_{jkl} - f_{ikl} = 2 \pi n. \quad (16)$$

This integrality condition will not concern us in our Lagrangian formulation but it has to be imposed as an external condition.

Two triples, represented by  $(B, A, f)$  and  $(B', A', f')$  respectively, are gauge equivalent if they satisfy the relations

$$B'_i = B_i + dC_i, \quad \text{on } U_i \quad (17)$$

$$A'_{ij} = A_{ij} + C_i - C_j + d\lambda_{ij} \quad \text{on } U_i \cap U_j \quad (18)$$

$$f'_{ijk} = f_{ijk} + \lambda_{ij} + \lambda_{ki} + \lambda_{jk} \quad \text{on } U_i \cap U_j \cap U_k \quad (19)$$

for the one-forms  $C$  and the zero-forms  $\lambda$ .

The pattern for the higher order gerbes is rather clear. For instance, the 2-gerbes will be characterized by a quadruple starting from a 3-form and going down to a 0-form field, etc.

We want now to transfer this geometrical information to field theory. The field content of a 1-gerbe is clear: it is made up of a two-form field  $B$ , a one-form gauge field  $A$  and a scalar field  $f$  with the gauge transformations

$$\delta B = dC, \quad \delta C = C + d\lambda, \quad \delta f = \lambda. \quad (20)$$

---

<sup>4</sup>Henceforth, it will be convenient to use the more synthetic language of forms, rather than the component fields, which have been used earlier in the text.

## 5 (anti-)BRST for gerbes

We wish to define the BRST and anti-BRST transformations for the above theory. The most general field content is given by the triple  $(B, A, f)$ . But since  $f$  has 0 canonical dimension and since the essential features are contained in the couple  $(B, A)$ , we will consider here only the latter. The inclusion of  $f$  is not difficult but yields more cumbersome formulas. Let us start from a table that contains the order form and ghost number of all the fields involved:

field	$B$	$A$	$K$	$\bar{K}$	$C$	$\bar{C}$	$\beta$	$\bar{\beta}$	$\lambda$	$\bar{\lambda}$	$\rho$	$\bar{\rho}$	$g$	$\bar{g}$
form order	2	1	1	1	1	1	0	0	0	0	0	0	0	0
ghost number	0	0	0	0	1	-1	2	-2	1	-1	1	-1	0	0

The appropriate BRST and anti-BRST transformations turn out to be

$$\begin{aligned}
s_b B &= dC, & s_b C &= -d\beta, \\
s_b A &= C + d\lambda, & s_b \lambda &= \beta, \\
s_b \bar{C} &= -K, & s_b \bar{K} &= d\rho, \\
s_b \bar{\beta} &= -\bar{\rho}, & s_b \bar{\lambda} &= g, & s_b \bar{g} &= \rho,
\end{aligned} \tag{21}$$

together with  $s_b[\rho, \bar{\rho}, g, K_\mu, \beta] = 0$ , and

$$\begin{aligned}
s_{ab} B &= d\bar{C}, & s_{ab} \bar{C} &= +d\bar{\beta}, \\
s_{ab} A &= \bar{C} + d\bar{\lambda}, & s_{ab} \bar{\lambda} &= -\bar{\beta}, \\
s_{ab} C &= +\bar{K}, & s_{ab} K &= -d\bar{\rho}, \\
s_{ab} \beta &= +\rho, & s_{ab} \lambda &= -\bar{g}, & s_{ab} g &= -\bar{\rho},
\end{aligned} \tag{22}$$

while  $s_{ab}[\bar{\beta}, \bar{g}, \bar{K}_\mu, \rho, \bar{\rho}] = 0$ .

It can be easily verified that  $(s_b + s_{ab})^2 = 0$  if the following constraint is satisfied:

$$\bar{K}_\mu - \partial_\mu \bar{g} = K_\mu - \partial_\mu g. \tag{23}$$

This condition is both BRST and anti-BRST invariant. It is the analogue of the constraint (9) above and the analogue of the Curci-Ferrari condition in non-Abelian gauge theories.

It is also evident that, if we disregard the potential  $A$ , the transformations (21,22) reduce to (7,8). Therefore the latter is but a particular case of the transformations introduced in this section. Actions with the symmetry (21,22) as well as the implications with the superfield formalism [8, 9, 4, 10] will be analyzed elsewhere.



In the case of a 2-gerbe with field content  $(C, B, A, f)$  with order form  $(3, 2, 1, 0)$  respectively (and ghost number zero), it is not hard to verify that in order to satisfy  $(s_b + s_{ab})^2 = 0$  one has to impose two constraints

$$H - \bar{H} + d(K - \bar{K}) = 0, \quad \bar{K} - K = d(\bar{g} - g). \quad (24)$$

where  $(H, K, g)$  as well as the corresponding barred fields are  $(2, 1, 0)$ -form field, respectively, with ghost number 0. It is not hard to imagine how this will generalize to higher order gerbes. This shows, in particular, that such constraints as (9, 23, 24) are strictly linked to the gerbe structure.

## 6 Discussion

The condition  $(s_b + s_{ab})^2 = 0$  is a condition that one should always require. We recall the geometrical interpretation of the BRST transformation in [11]. In non-Abelian gauge theories a BRST transformation is just an alias for the set of all the gauge transformations. The nilpotency of  $s_b$  represents the consistency which is required upon doing two gauge transformations in different orders. The anti-BRST transformation represents an independent version of the same operation, therefore it must be nilpotent too. But considered together, a BRST and an anti-BRST are just another way to represent the set of gauge transformations. Therefore they must satisfy collective nilpotency, i.e.  $s_b + s_{ab}$  must be nilpotent, so that, in particular,  $s_b s_{ab} + s_{ab} s_b = 0$ .

This interpretation holds also for the transformations considered in this paper. It follows that the constraints (9, 23, 24) have to be imposed for consistency. The question that remains to be clarified is their geometrical meaning, if any. In [11] the Curci-Ferrari constraints for non-Abelian gauge theories were put in the appropriate geometrical context, but a geometrical interpretation is still lacking. We do not have a coherent geometrical interpretation of (9, 23, 24) either. However we would like to make some remarks.

First, looking at (9) we notice that it defines a De Rham cohomology class, represented by the one-form  $B_\mu$ . Second,  $\phi$  is a nontrivial cocycle of  $s_b + s_{ab}$ . Third  $\phi$  appears in degree two starting from  $B_{\mu\nu}$ . Similar things can be said about (23), changing  $B_\mu$  with  $K_\mu$  and  $\phi$  with  $g - \bar{g}$ . Therefore (9, 23), and likewise (24), look like transgression relations. It would be very interesting to obtain a complete picture of the geometry behind these relations.

## Acknowledgements

L.B. would like to acknowledge a useful discussion he had with E.Aldrovandi concerning gerbes. R.P.M. would like to thank SISSA, Trieste, Italy for the warm hospitality extended to him during his stay at SISSA. This research was supported for L.B. by the Italian MIUR under the program “Superstringhe, Brane e Interazioni Fondamentali”.

## References

- [1] M. Henneaux, C. Teitelboim, *Quantization of fields with constraints*, Princeton University Press, Princeton (NJ), 1992.
- [2] E. Harikumar, R. P. Malik, M. Sivakumar, *Hodge decomposition theorem for Abelian two-form gauge theory*, J. Phys. A: Math. Gen. **33** (2000) 7149 [arXiv: hep-th/0004145].
- [3] R. P. Malik, *Abelian 2-form gauge theory: special features*, J. Phys. A: Math. Gen. **36** (2003) 5095 [arXiv: hep-th/0209136].
- [4] R. P. Malik, *Superfield approach to nilpotent (anti-)BRST symmetry transformations for the Abelian 2-form gauge theory* [arXiv: hep-th/0702039].
- [5] P. A. M. Dirac, *Lectures on quantum mechanics*, Belfer Graduate School of Science, Yeshiva University Press, New York, 1964.
- [6] K. Sundermeyer, *Constrained dynamics: Lecture notes in physics*, vol. 169, Springer-Verlag, Berlin, 1982.
- [7] G. Curci, R. Ferrari, *Slavnov transformations and supersymmetry*, Phys. Lett. **B 63** (1976) 51.
- [8] L. Bonora, M. Tonin, *Superfield formulation of extended BRS symmetry*, Phys. Lett.**B 98** (1981) 48.
- [9] L. Bonora, P. Pasti, M. Tonin, *Geometric description of extended BRS symmetry in superfield formulation*, Nuovo Cimento **A 63** (1981) 353.
- [10] R. P. Malik, *Nilpotent symmetry invariance in the superfield formalism: the 1-form (non-)Abelian gauge theories* [arXiv: hep-th/0704.0064].
- [11] L.Bonora, P.Cotta-Ramusino, *Some remarks on BRS transformations, anomalies and the cohomology of the Lie algebra of the group of gauge transformations*, Comm.Math.Phys. **87** (1983) 589.

- [12] J. Giraud, *Cohomologie non-abélienne*, Grundle. **179**, Springer Verlag, Berlin (1971).
  - [13] J.-L. Brylinski, *Characteristic classes and geometric quantization*, Progr. in Math. **107**, Birkhäuser, Boston (1993).
  - [14] N. Hitchin, *Lectures on special Lagrangian submanifolds*, [arXiv:math/9907034]
  - [15] A. L. Carey, J. Mickelsson and M. K. Murray, *Bundle gerbes applied to quantum field theory*, Rev. Math. Phys. **12** (2000) 65 [arXiv:hep-th/9711133].
- J. Kalkkinen, *Gerbes and massive type II configurations*, JHEP **9907** (1999) 002 [arXiv:hep-th/9905018].
- P. Bouwknegt, A. L. Carey, V. Mathai, M. K. Murray and D. Stevenson, *Twisted K-theory and K-theory of bundle gerbes*, Commun. Math. Phys. **228** (2002) 17 [arXiv:hep-th/0106194].
- M. I. Caicedo, I. Martin and A. Restuccia, *Gerbes and duality*, Annals Phys. **300** (2002) 32 [arXiv:hep-th/0205002].
- K. Gawedzki and N. Reis, *WZW branes and gerbes*, Rev. Math. Phys. **14** (2002) 1281 [arXiv:hep-th/0205233].
- P. Aschieri, L. Cantini and B. Jurco, *Nonabelian bundle gerbes, their differential geometry and gauge theory*, Commun. Math. Phys. **254** (2005) 367 [arXiv:hep-th/0312154].
- J. M. Isidro, *Gerbes and Heisenberg's uncertainty principle*, Int. J. Geom. Meth. Mod. Phys. **3** (2006) 1469 [arXiv:hep-th/0512241].